การสร้างตัวแบบเชิงคณิตศาสตร์ของการตอบสนองทางสภาพต้านทานแบบแมกนิโทเมตริกจากตัวกลางวิวิธพันธุ์ซึ่งอยู่ใต้พื้นผิวดินเอกพันธุ์

Mathematical Modelling of Magnetometric Resistivity Response from a Heterogeneous Medium beneath a Homogeneous Overburden

วรินทร์ ศรีปัญญา

สาขาวิชาคณิตศาสตร์ คณะวิทยาศาสตร์และเทคโนโลยี มหาวิทยาลัยราชภัฏนครปฐม
ศูนย์วิจัยคณิตศาสตร์บริสุทธิ์และประยุกต์ สถาบันวิจัยและพัฒนา มหาวิทยาลัยราชภัฏนครปฐม
w.sripanya@windowslive.com

บทคัดย่อ

ผลเฉลยเชิงวิเคราะห์ของสนามแม่เหล็กในสภาวะคงตัวอันเป็นผลมาจากแหล่งกำเนิดไฟฟ้ากระแสตรงได้ถูกนำเสนอขึ้นสำหรับปัญหาของแหล่งจ่ายไฟฟ้าและตัวรับสัญญาณซึ่งถูกฝังอยู่ในพื้นผิวดินวิวิธพันธุ์ 2 ประเภท ตัวแบบของพื้นโลกที่มีลักษณะเป็น 2 ชั้นต่อเนื่องกันถูกนำมาพิจารณาในกรณีที่พื้นผิวดินเอกพันธุ์อยู่เหนือตัวกลางที่มีสภาพนำไฟฟ้าเปลี่ยนแปลงแบบเชิงกำลังและแบบเชิงเส้นตามความลึกของพื้นโลก การแปลงฮันเกลแบบทั่วไปถูกนำมาใช้ในปัญหาซึ่งทำให้ได้ผลเฉลยเชิงวิเคราะห์ ผลลัพธ์ที่เกิดขึ้นถูกนำมาเขียนกราฟแสดงพฤติกรรมของสนามแม่เหล็ก แนวเส้นโค้งของสนามแม่เหล็กแสดงถึงนัยสำคัญบางประการของการแปรผันของสภาพนำไฟฟ้า

คำสำคัญ: สภาพต้านทานแบบแมกนิโทเมตริก ตัวกลางวิวิธพันธุ์ การแปลงฮันเกล

Abstract

Analytical solutions of the steady state magnetic field resulting from a direct current source are derived for the problem of a buried current source and a buried receiver within two types of heterogeneous earth structures. Two-layered continuously earth models are considered in the cases of a homogeneous overburden overlying the host media whose conductivities vary exponentially and linearly with depth. The generalized Hankel transform is introduced to our problem and analytical results are obtained. The effects of magnetic fields are plotted to show the behavior of the fields. The curves of magnetic fields show some significance to the variation of conductivity.

Keywords: magnetometric resistivity, heterogeneous medium, Hankel transform

1. Introduction

Many authors have investigated the nature of the resistivity response resulting from a heterogeneous ground whose electrical conductivities vary continuously with depth. Stoyer and Wait (1977) firstly considered the electrical conductivity varying exponentially with depth under homogeneous overburden. Banerjee et al. (1980), Kim and Lee (1996) discussed the problem of a multilayered earth and derived the specific case for a two-layered model. In transitional layers, the electrical conductivity is assumed, for simplicity, to be linearly dependent upon depth. The problem was first treated by Mallick and Roy (1968) that presented an analysis of the problem of a two-layered earth. Koefoed (1979) solved the problem with a linear change of the resistivity with depth, a type of change that seems to be more common in nature than the type considered by Mallick and Roy. Banerjee et al. (1980) studied the electrical conductivity in a transitional layer which is assumed to be binomially with depth and derived some particular cases of a linear change.

In this paper, we derive analytical solutions of the steady state magnetic field resulting from a direct current source buried within two types of two-layered heterogeneous earth structures with a homogeneous overburden in which the electrical conductivities in a host medium vary exponentially and linearly with depth. The generalized Hankel transform is introduced to our problem and analytical results are obtained.

2. Magnetic Field from a DC Source in a 1D Structure

Let us consider a geometric model of the earth's structure described and discussed by Sato (2000). The earth's structure consists of two conductive half-spaces. The half-space above the ground surface $\left(z<0\right)$ is a region of air, denoted by layer 0. The half-space below the ground surface $\left(z>0\right)$ is a two-layered horizontally stratified earth with depth to the layer h (the lowermost layer extending to infinity) measured from the ground surface. A point source of direct current I is deliberately located at the interface $z=h$ of layer 1 and layer 2 to simplify the mathematics. Each layer has conductivity as a function of depth, i.e., $\sigma_k\left(z\right)$ for layer $0\le k\le 2$. The general steady state Maxwell's equations in the frequency domain (Edwards, 1988) can be used to determine the magnetic field for this problem. Since the problem is axisymmetric and the vector magnetic field has only the azimuthal component in cylindrical coordinates, for simplicity, we use $\tilde{H}$ to represent the azimuthal component in the following derivations (Sripanya, 2016), and we now have

$$\frac{\partial^2\tilde{H}}{\partial z^2}+\sigma\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\right)\frac{\partial\tilde{H}}{\partial z}-\lambda^2\tilde{H}=0\,, \tag{1}$$

The magnetic field in each layer can be obtained by taking the inverse generalized Hankel transform (Ali and Kalla, 1999) to the solution of equation (1). The problem satisfies the physical boundary conditions presented by Sripanya (2016).

3. Solution of the Problem

3.1 Exponential Profile

For an exponentially varying conductivity profile, the variation of conductivity is denoted by

$$\sigma\left(z\right) = a\exp\left(b\left(z-h\right)\right), \tag{2}$$

where a, b and h are constants that preserve $\sigma\left(z\right) > 0$. Hence, the magnetic field in an exponentially varying conductive ground can be written as

$$\tilde{H}\left(\lambda, z\right) = A\exp\left(\left(z-h\right)\xi\right) + B\exp\left(\left(z-h\right)\zeta\right), \tag{3}$$

where ξ and ζ are given by

$$\xi = \frac{1}{2}\left(b - \sqrt{b^2+4\lambda^2}\right) \quad \text{and} \quad \zeta = \frac{1}{2}\left(b + \sqrt{b^2+4\lambda^2}\right). \tag{4}$$

The unknown coefficients A and B are arbitrary constants, which can be determined by using the boundary conditions (see Sato, 2000).

3.2 Linear Profile

For a linearly varying conductivity profile, the variation of conductivity is denoted by

$$\sigma\left(z\right) = a + m\left(z-h\right), \tag{5}$$

where a, h and $m \neq 0$ are constants that preserve $\sigma\left(z\right) > 0$. Hence, the magnetic field in a linearly varying conductive ground can be written as

$$\tilde{H}\left(\lambda, z\right) = \vartheta\left(z\right)\left(CI_1\left(\frac{\lambda}{\varphi}\vartheta\left(z\right)\right) + DK_1\left(\frac{\lambda}{\varphi}\vartheta\left(z\right)\right)\right), \tag{6}$$

where φ and ϑ are given by

$$\varphi = m/a \quad \text{and} \quad \vartheta\left(z\right) = 1 + \varphi\left(z-h\right). \tag{7}$$

The special functions I_1 and K_1 are the modified Bessel functions of the first and second kinds of order one, respectively, and the unknown coefficients C and D are arbitrary constants, which can be determined by using the boundary conditions.

4. 2-layered Earth Model

Consider a 2-layered earth model with a nonconductive layer 0 (representing a region of air). An overburden has a constant conductivity a with thickness h overlying a host medium having continuously varying conductivity σ, as given above, with infinite depth. A current electrode is located at the interface $z = h$.

4.1 Exponential Profile

The magnetic field in an exponentially varying conductive medium can be written as

$$H\left(r,z\right) = \int_0^\infty \frac{I}{2\pi}\left(\frac{\lambda\cosh\left(\lambda h\right)\exp\left(\left(z-h\right)\xi\right)}{\lambda\cosh\left(\lambda h\right)-\sinh\left(\lambda h\right)\xi}\right)J_1\left(\lambda r\right)d\lambda\,, \tag{8}$$

where J_1 is the Bessel function of the first kind of order one.

4.2 Linear Profile

The magnetic field in a linearly varying conductive medium can be written as

$$H\left(r,z\right) = \int_0^\infty \frac{I}{2\pi}\left(\frac{\vartheta\left(z\right)K_1\left(\left(\lambda/\varphi\right)\vartheta\left(z\right)\right)}{K_1\left(\lambda/\varphi\right)+\tanh\left(\lambda h\right)K_0\left(\lambda/\varphi\right)}\right)J_1\left(\lambda r\right)d\lambda\,, \tag{9}$$

where K_0 is the modified Bessel function of the second kind of order zero.

5. Numerical Experiments

In our sample tests, we calculate the magnetic fields resulting from the models of practical interest. Both of the example models are 2-layered electrically conductive earth structures with a nonconductive layer 0, as shown in Section 4. These models are referred to as Models 1 and 2, which are given in equations (8) and (9), respectively. The values of the model parameters are tabulated in Table 1. Chave's algorithm (1983) is used for numerically calculating the inverse Hankel transform of the magnetic field solutions. The special functions are computed by using the Numerical Recipes source codes (Press et al., 1992). The electric current of 1 ampere is used in our computations. The results of our models are plotted and compared to show the behavior of magnetic fields against source-receiver spacing r at different depths $z = 5, 5.5, \ldots, 15$ metres as shown in Figure 1. We see that the curves of magnetic fields obtained from the first model are quite different from the second model. The contrast of electrical conductivity gives effect to the magnitude of magnetic field. It can be seen that the magnitude of magnetic field from the second model is much higher than the magnetic field from the first model at the same depth and source-receiver spacing. We observe that if the conductivity of a host medium is high, it will lead to the large sized magnitude of magnetic field. This means that the differences of magnetic curves are depended on the variation of conductivity.

Table 1 Model parameters used in our sample tests

Model	a (S/m)	b (m^{-1})	m (S/m^2)	h (m)
1	0.1	0.25	-	5
2	0.1	-	0.1	5

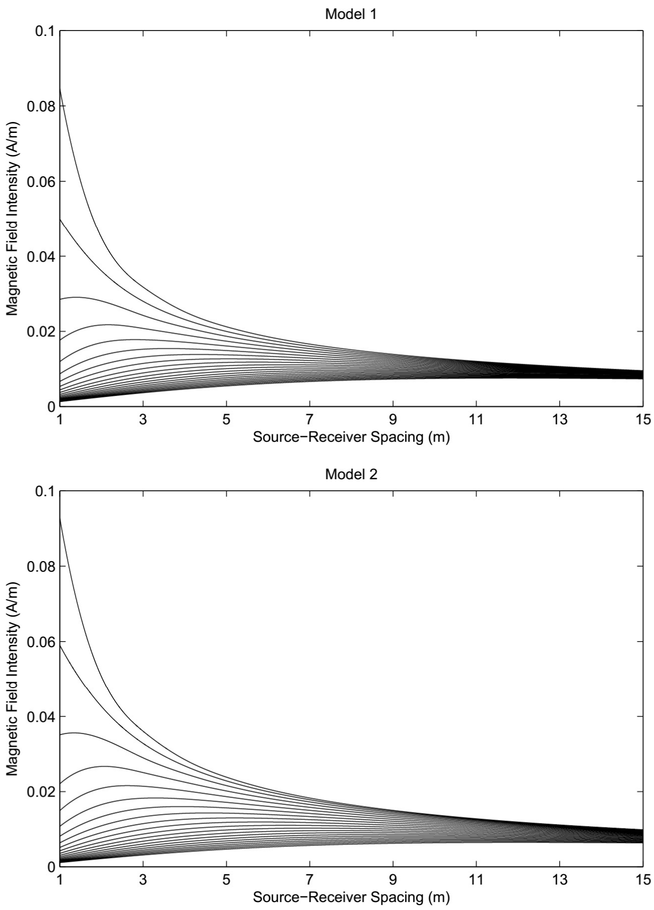

Figure 1 Behavior of magnetic fields from our example models.

6. Conclusions

We have derived analytical solutions of the steady state magnetic field for the problem of source and receiver electrodes buried within two types of two-layered heterogeneous earth structures with a homogeneous overburden in which the electrical conductivities in a host medium vary exponentially and linearly with depth. The generalized Hankel transform is introduced to our problem and analytical results are obtained. The effects of magnetic fields are plotted and compared to show the behavior in response to different ground structures at many depths while some parameters are approximately given. The curves of magnetic fields show some significance to the variation of conductivity.

7. References

Ali, I., & Kalla, S. (1999). A generalized Hankel transform and its use for solving certain partial differential equations. **J. Austral. Math. Soc. Ser. B.**, 41 (1), 105-117.

Banerjee, B., Sengupta, B. J., & Pal, B. P. (1980). Apparent resistivity of a multilayered earth with a layer having exponentiality varying conductivity. **Geophys. Prosp.**, 28 (3), 435-452.

Banerjee, B., Sengupta, B. J., & Pal, B. P. (1980). Resistivity sounding on a multilayered earth containing transition layers. **Geophys. Prosp.**, 28 (5), 750-758.

Chave, A. D. (1983). Numerical integration of related Hankel transforms by quadrature and continued fraction expansion. **Geophysics.**, 48 (12), 1671-1686.

Edwards, R. N. (1988). A downhole magnetometric resistivity technique for electrical sounding beneath a conductive surface layer. **Geophysics.**, 53 (4), 528-536.

Kim, H.-S., & Lee, K. (1996). Response of a multilayered earth with layers having exponentially varying resistivities. **Geophysics.**, 61 (1), 180-191.

Koefoed, O. (1979). Resistivity sounding on an earth model containing transition layers with linear change of resistivity with depth. **Geophys. Prosp.**, 27 (4), 862-868.

Mallick, K., & Roy, A. (1968). Resistivity sounding on a two-layer earth with transitional boundary. **Geophys. Prosp.**, 16 (4), 436-446.

Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. (1992). **Numerical Recipes in Fortran 77: The Art of Scientific Computing, Fortran Numerical Recipes Vol. 1** (2nd ed.). Cambridge Univ. Press.

Sato, H. K. (2000). Potential field from a dc current source arbitrarily located in a nonuniform layered medium. **Geophysics.**, 65 (6), 1726-1732.

Sripanya, W. (2016). Mathematical modelling of magnetic field from a DC source in a multilayered medium. **J. Interdiscip. Math.**, 19 (5&6), 1015-1023.

Stoyer, C. H., & Wait, J. R. (1977). Resistivity probing of an “exponential” earth with a homogeneous overburden. **Geoexpl.**, 15 (1), 11-18.