Testing the Coefficient of Variation for the Inverse Gamma Distribution: A Case Study of the Annual Rainfall Amounts in Lampang, Thailand

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Abstract

Two test statistics for testing the coefficient of variation in an inverse gamma distribution were proposed in this study. The proposed test statistics were based on the score and Wald methods. An evaluation of the performance of the proposed test statistics using Monte Carlo simulations was conducted under several shape parameter values in an inverse gamma distribution. The performances of the test statistics were compared based on the empirical type I error rates and the powers of the tests. The simulation results revealed that the test statistics based on the Wald method performed better than the test statistics based on the score method in terms of the attained nominal significance level and is thus recommended for analysis in similar scenarios. The efficacies of the proposed test statistics were also illustrated by applying them to annual rainfall amounts in Lampang, Thailand.

Keywords: statistical test, measure of dispersion, continuous distribution, type I error rate, powers of the test

1. Introduction

The coefficient of variation (CV) is a unit-free measure of variability relative to the population mean [1]. It is defined as the ratio of the population standard deviation σ to the population mean μ, namely $\theta = \sigma / \mu$, where $\mu \neq 0$. It has been more widely used than the standard deviation for comparing the variations of several variables obtained by different units. The estimator of the CV has been widely applied in many fields of science, including the medical sciences, engineering, economics and others (see Nairy and Rao [2]). For example, the applicability of the CV method for analyzing synaptic plasticity was studied by Faber and Korn [3]. Calif and Soubdhan [4] used the CV to measure the spatial and temporal correlation of global solar radiation. Reed et al. [5] used the CV in assessing the variability of quantitative assays. Bedeian and Mossholder [6] used the CV for comparing diversity in work groups. Kang et al. [7] applied the CV for monitoring variability in statistical process control. Castagliola et al. [8] proposed a new method to monitor the CV by means of two one-sided exponentially weighted moving average charts of the CV squared. Döring and Reckling [9] proposed a method to adjust the standard CV to account for the systematic dependence of population variance from the population mean.

In probability and statistics, the inverse gamma distribution is a two-parameter family of continuous distributions on the positive real line, which is the distribution of the reciprocal of a variable distributed according to the gamma distribution [10]. The inverse gamma distribution is most often used as a conjugate prior distribution in Bayesian statistics. There are several research papers to study the distribution of the inverse gamma. For example, Gelman [11] applied inverse gamma distribution as the prior distributions for variance parameters in hierarchical models. Abid and Al-Hassany [10] studied some issues related with inverted gamma distribution which is the reciprocal of the gamma distribution. Llera and Beckmann [12] introduced five different algorithms based on method of moments, maximum likelihood method and Bayesian method to estimate the parameters of inverted gamma distribution. Glen and Leemis [13] studied the inverse gamma distribution as a survival distribution.

The literature on testing the CV for the inverse gamma distribution is limited. However, there are many methods available for estimating the confidence interval for a population CV of the inverse gamma distribution. Kaewprasert et al. [14] presented three confidence intervals for the CV of the inverse gamma distribution using the score method, the Wald method and the percentile bootstrap confidence interval. These confidence intervals for the CV can be used to test the hypothesis for the CV.

The objective of this paper is to propose some methods for testing the CV for the inverse gamma distribution and identify the appropriate methods for practitioners. Two confidence intervals proposed by Kaewprasert et al. [14] are considered in order to test the population CV. A simulation study was conducted to compare the performance of these methods. Based on the simulation results, test statistics with high power that attained a nominal significance level are recommended for practitioners.

The structure of this paper is as follows. The point estimation of parameters in an inverse gamma distribution are reviewed in the Section 2. In Section 3, we present the proposed methods for testing the CV of the inverse gamma distribution. The simulation study and results are discussed in Section 4. Section 5 shows the application of the proposed statistical tests to real data is shown using the annual rainfall amounts in Lampang, Thailand. Discussion and conclusions are presented in the final section.

2. Point Estimation of Parameters in an Inverse Gamma Distribution

In this section, we explain the point estimation of parameters in an inverse gamma distribution. Let $X_1,...,X_n$ be a random sample from the inverse gamma distribution with the shape parameter α and scale parameter β, denoted as $IG(\alpha,\beta)$. The probability density function of X is given by

$$f(x;\alpha,\beta)=\frac{\beta^{\alpha}}{\Gamma(\alpha)}x^{-\alpha-1}\exp\left(-\frac{\beta}{x}\right),\quad x>0,\ \alpha>0,\ \beta>0. \tag{1}$$

The population mean and variance of X are defined as

$$E(X)=\frac{\beta}{\alpha-1},\quad \text{for } \alpha>1$$

and

$$Var(X)=\frac{\beta^2}{(\alpha-1)^2(\alpha-2)},\quad \text{for } \alpha>2.$$

Therefore, the CV of X can be expressed as

$$CV(X)=\theta=\frac{1}{\sqrt{\alpha-2}}.$$

Since α is an unknown parameter, it is required to be estimated. The maximum likelihood estimators (MLE) for α and β are considered. From the probability density function shown in (1), the log-likelihood function of α and β is given by

$$\ln L(\alpha,\beta)=-\sum_{i=1}^{n}\left(\frac{\beta}{X_i}\right)-(\alpha+1)\sum_{i=1}^{n}\ln(X_i)-n\ln\Gamma(\alpha)+n\alpha\ln(\beta).$$

Taking partial derivatives of the above equation with respect to α and β, respectively, the score function is derived as

$$U(\alpha,\beta)=\begin{bmatrix}\sum_{i=1}^{n}\ln(X_i)-\text{n}\ln(\alpha)+\frac{n}{2\alpha}-n\ln(\beta)\\ -\sum_{i=1}^{n}X_i^{-1}+\frac{n\alpha}{\beta}\end{bmatrix}.$$

Then, the maximum likelihood estimators can be conducted for α and β, respectively,

$$\hat{\alpha}=\frac{1}{2\left[\frac{\sum_{i=1}^{n}\ln(X_i)}{n}+\ln\left(\frac{\sum_{i=1}^{n}X_i^{-1}}{n}\right)\right]},\quad \text{and}\quad \hat{\beta}=\frac{n\hat{\alpha}}{\sum_{i=1}^{n}X_i^{-1}}.$$

Also, the estimator of CV is given by $\hat{\theta}=\frac{1}{\sqrt{\hat{\alpha}-2}}$.

3. Methods for Testing the Coefficient of Variation of the Inverse Gamma Distribution

Let $X_1,\ldots,X_n$ be an independent and identically distributed (i.i.d.) random sample of size n from the inverse gamma distribution with the shape parameter α and scale parameter β. We want to test for the population CV. The null and alternative hypotheses are defined as follows:

$$H_0:\theta=\theta_0 \quad \text{versus} \quad H_1:\theta\neq\theta_0.$$

In this section, we discuss two test statistics for the CV based on the score method and the Wald method.

1) Score method

Let α and β be the parameter of interest and the nuisance parameters, respectively. In general, the score statistic is denoted as

$$W_1 = U^T(\alpha_0, \hat{\beta}_0) I^{-1}(\alpha_0, \hat{\beta}_0) U(\alpha_0, \hat{\beta}_0),$$

where $\hat{\beta}_0$ is the maximum likelihood estimator for β, under the null hypothesis $H_0' : \alpha = \alpha_0$, $U(\alpha_0, \hat{\beta}_0)$ is the vector of the score function and $I(\alpha_0, \hat{\beta}_0)$ is the matrix of the Fisher information. Here, it is easy to derive that the score function under H_0' is

$$U(\alpha_0, \hat{\beta}_0) = \begin{bmatrix} -\sum_{i=1}^{n} \ln(X_i) + \dfrac{n}{2\alpha_0} - n \ln\left(\dfrac{n}{\sum_{i=1}^{n} X_i^{-1}} \right) \\ 0 \end{bmatrix}.$$

The inverse of the matrix of the Fisher information can be derived as follows

$$I^{-1}(\alpha_0, \hat{\beta}_0) = \begin{bmatrix} \dfrac{2\alpha_0^2}{n} & -\dfrac{2\alpha_0^2}{\sum_{i=1}^{n} X_i^{-1}} \\ -\dfrac{2\alpha_0^2}{\sum_{i=1}^{n} X_i^{-1}} & \dfrac{n\alpha_0(2\alpha_0 - 1)}{\left(\sum_{i=1}^{n} X_i^{-1} \right)^2} \end{bmatrix}.$$

Using the property of the score function, we can see that the pivotal

$$Z_{score} = \sqrt{\frac{2\alpha_0^2}{n}} \left[-\sum_{i=1}^{n} \ln(X_i) + \frac{n}{2\alpha_0} + n \ln\left(\frac{n}{\sum_{i=1}^{n} X_i^{-1}} \right) \right] \qquad (2)$$

converges in distribution to the standard normal distribution. Since the variance of $\hat{\alpha}$ is $\frac{2\alpha_0^2}{n}$, it is approximated by substituting $\hat{\alpha}$ in its variance. Under H_0', the statistic in (2) is given as

$$Z_{score} \cong \sqrt{\frac{2\hat{\alpha}^2}{n}} \left[-\sum_{i=1}^{n} \ln(X_i) + \frac{n}{2\hat{\alpha}} + n \ln\left(\frac{n}{\sum_{i=1}^{n} X_i^{-1}} \right) \right].$$

From the probability statement,

$$1 - \gamma = P(-Z_{1-\gamma/2} \le Z_{score} \le Z_{1-\gamma/2}),$$

it can be simply written as

$$1 - \gamma = P(l_s \le \theta \le u_s).$$

Therefore, the $(1-\gamma)100\%$ confidence interval for θ based on the score method is given by

$$CI_S = [l_s, u_s] = \left[\frac{1}{\sqrt{\dfrac{n}{2\left(z_1 - Z_{\gamma/2}\sqrt{\dfrac{n}{2\hat{\alpha}^2}} \right)} - 2}}, \frac{1}{\sqrt{\dfrac{n}{2\left(z_1 + Z_{\gamma/2}\sqrt{\dfrac{n}{2\hat{\alpha}^2}} \right)} - 2}} \right],$$

where $z_1 = \sum_{i=1}^{n} \ln(X_i) - n \ln\left(\frac{n}{\sum_{i=1}^{n} X_i^{-1}} \right)$ and $Z_{\gamma/2}$ is the $\gamma/2$-upper quantile of the standard normal distribution.

Therefore, we will reject the null hypothesis, $H_0 : \theta = \theta_0$, if

$$\theta_0 < \frac{1}{\sqrt{\frac{n}{2\left(z_1 - Z_{\gamma/2}\sqrt{\frac{n}{2\hat{\alpha}^2}}\right)} - 2}} \quad \text{or} \quad \theta_0 > \frac{1}{\sqrt{\frac{n}{2\left(z_1 + Z_{\gamma/2}\sqrt{\frac{n}{2\hat{\alpha}^2}}\right)} - 2}}.$$

2) Wald method

The Wald statistic is an asymptotic statistic derived from the property of the maximum likelihood estimator. The general form of the Wald statistic under the null hypothesis $H_0' : \alpha = \alpha_0$ is defined as

$$W_2 = (\hat{\alpha} - \alpha_0)^T \left[I^{\alpha\alpha}(\hat{\alpha}, \hat{\beta}) \right]^{-1} (\hat{\alpha} - \alpha_0),$$

where $I^{\alpha\alpha}(\hat{\alpha}, \hat{\beta})$ is the estimated variance of $\hat{\alpha}$ obtained from the first row and the first column of $I^{-1}(\hat{\alpha}, \hat{\beta})$. Using the information of partial derivatives from the previous subsection, the inverse matrix is given by

$$I^{-1}(\hat{\alpha}, \hat{\beta}) = \begin{bmatrix} \frac{2\hat{\alpha}^2}{n} & -\frac{2\hat{\alpha}^2}{\sum_{i=1}^{n} X_i^{-1}} \\ -\frac{2\hat{\alpha}^2}{\sum_{i=1}^{n} X_i^{-1}} & \frac{n\hat{\alpha}(2\hat{\alpha}-1)}{\left(\sum_{i=1}^{n} X_i^{-1}\right)^2} \end{bmatrix},$$

where $I^{\alpha\alpha}(\hat{\alpha}, \hat{\beta}) = \frac{2\hat{\alpha}^2}{n}$. Therefore, under H_0', we obtain the Wald statistic

$$Z_{wald} \cong \sqrt{\frac{n}{2\hat{\alpha}^2}} (\hat{\alpha} - \alpha), \quad (3)$$

which has the limiting distribution of a standard normal distribution. Thus, the $(1-\gamma)100\%$ confidence interval for θ based on the Wald method is given by

$$CI_W = [l_w, u_w] = \left[\frac{1}{\sqrt{\hat{\alpha} - 2 + Z_{\gamma/2}\sqrt{\frac{2\hat{\alpha}^2}{n}}}}, \frac{1}{\sqrt{\hat{\alpha} - 2 - Z_{\gamma/2}\sqrt{\frac{2\hat{\alpha}^2}{n}}}} \right],$$

where $Z_{\gamma/2}$ is the $\gamma/2$-upper quantile of the standard normal distribution. Therefore, we will reject the null hypothesis, $H_0 : \theta = \theta_0$, if

$$\theta_0 < \frac{1}{\sqrt{\hat{\alpha} - 2 + Z_{\gamma/2}\sqrt{\frac{2\hat{\alpha}^2}{n}}}} \quad \text{or} \quad \theta_0 > \frac{1}{\sqrt{\hat{\alpha} - 2 - Z_{\gamma/2}\sqrt{\frac{2\hat{\alpha}^2}{n}}}}.$$

4. Simulation Study and Results

In this study, two statistical methods for testing the population CV in an inverse gamma distribution are considered. Since a theoretical comparison is not possible, a Monte Carlo simulation was conducted using the R version 4.1.3 statistical software [15] to compare the performance of the test statistics. The methods were compared in terms of their attainment of empirical type I error rates and the powers of their performance. The simulation results are presented only for the significant level γ =0.05, since a) γ =0.05 is widely used to compare the power of the test and b) similar conclusions were obtained for other values of γ.

To observe the behavior of small, moderate and large sample sizes, we used n = 25, 50, 75, 100 and 200. The number of simulations was fixed at 10,000. The data were generated from an inverse gamma distribution with $\beta = 1$ and α was adjusted to obtain the required coefficient of variation θ. We set $\theta = 0.05$, 0.10, 0.15, 0.20, 0.25, 0.30 and 0.35.

As can be seen in the simulation results displayed in Tables 1-7, the empirical type I error rates of the Wald method were close to the nominal significance level of 0.05 for all sample sizes while those of the score method were close to the nominal significance level of 0.05 for larger sample sizes. The score method performed well in terms of the power of the test for $\theta < \theta_0$. On the other hand, the Wald method performed better for $\theta > \theta_0$. We observed a general pattern; when the sample size increases, the power of the test also increases and the empirical type I error rate approaches 0.05. Also the power increases as the value of the CV departs from the hypothesized value of the CV. It was observed that for large sample sizes, the performance of the test statistics did not differ greatly in the sense of power and the attainment of the nominal significance level of the test. However, a significant difference was observed for small sample sizes.

Table 1. Empirical type I error rates (under line) and powers of tests for IG(402, 1), $\theta = 0.05$.

n	Method	θ_0								
		0.01	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
25	Score	1.0000	0.9974	0.6767	0.0568	<u>0.1125</u>	0.4796	0.8607	0.9850	0.9989
	Wald	1.0000	1.0000	0.9423	0.4102	<u>0.0383</u>	0.1088	0.4081	0.7516	0.9406
50	Score	1.0000	1.0000	0.9887	0.3329	<u>0.0790</u>	0.6190	0.9738	0.9998	1.0000
	Wald	1.0000	1.0000	0.9983	0.6442	<u>0.0446</u>	0.3042	0.8535	0.9943	0.9999
75	Score	1.0000	1.0000	0.9998	0.5840	<u>0.0742</u>	0.7498	0.9956	1.0000	1.0000
	Wald	1.0000	1.0000	1.0000	0.8026	<u>0.0472</u>	0.4971	0.9747	1.0000	1.0000
100	Score	1.0000	1.0000	1.0000	0.7624	<u>0.0693</u>	0.8292	0.9997	1.0000	1.0000
	Wald	1.0000	1.0000	1.0000	0.8954	<u>0.0485</u>	0.6434	0.9973	1.0000	1.0000
200	Score	1.0000	1.0000	1.0000	0.9838	<u>0.0618</u>	0.9739	1.0000	1.0000	1.0000
	Wald	1.0000	1.0000	1.0000	0.9938	<u>0.0484</u>	0.9406	1.0000	1.0000	1.0000

5. An Empirical Application

To illustrate the application of the two statistical methods for testing the CV introduced in the previous section, we used data on the annual rainfall amounts (millimeter: mm.) obtained from Upper Northern Region Irrigation Hydrology Center, Royal Irrigation Department, Thailand (https://www.hydro-1.net). The annual rainfall amounts were measured from the station at Kiew Lom Dam, Mueang District, Lampang, Thailand from 1992 to 2016. The descriptive statistics are as follows: sample size = 25, mean = 1186.97, standard deviation (SD) = 267.33, CV = 0.225, coefficient of skewness = 0.381, and kurtosis = -0.575. The distribution of the annual rainfall amount is slightly right-skewed and it has light tailed data distribution.

Table 2. Empirical type I error rates (under line) and powers of tests for IG(102, 1), $\theta = 0.10$.

n	Method	θ_0								
		0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14
25	Score	0.6696	0.2744	0.061	0.0413	0.1155	0.2711	0.4674	0.6806	0.8383
	Wald	0.9417	0.7357	0.3994	0.1433	0.0431	0.0483	0.1055	0.2268	0.3772
50	Score	0.9853	0.7963	0.3257	0.0583	0.0820	0.2996	0.6087	0.8623	0.9722
	Wald	0.9977	0.9455	0.6460	0.2192	0.0432	0.0962	0.2933	0.5913	0.8365
75	Score	0.9996	0.9561	0.5740	0.1078	0.0680	0.3319	0.7339	0.9436	0.9948
	Wald	1.0000	0.9883	0.7927	0.2888	0.0420	0.1454	0.4792	0.8088	0.9681
100	Score	1.0000	0.9911	0.7512	0.1785	0.0688	0.3806	0.8169	0.9816	0.9995
	Wald	1.0000	0.9982	0.8867	0.3592	0.0469	0.1976	0.6333	0.9316	0.9967
200	Score	1.0000	1.0000	0.9799	0.4410	0.0595	0.5595	0.9690	0.9999	1.0000
	Wald	1.0000	1.0000	0.9919	0.5863	0.0496	0.4089	0.9279	0.9992	1.0000

Table 3. Empirical type I error rates (under line) and powers of tests for IG(46.44, 1), $\theta = 0.15$.

n	Method	θ_0								
		0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19
25	Score	0.1695	0.0592	0.0323	0.0542	0.1131	0.1966	0.3222	0.4602	0.5987
	Wald	0.6244	0.3933	0.1982	0.0933	0.0398	0.035	0.0559	0.1014	0.1703
50	Score	0.6296	0.3168	0.0985	0.0401	0.0794	0.1973	0.3845	0.5925	0.7685
	Wald	0.8673	0.6352	0.3253	0.1241	0.0476	0.0580	0.1421	0.2815	0.4585
75	Score	0.8726	0.5564	0.2101	0.0549	0.0702	0.2085	0.4603	0.7160	0.8807
	Wald	0.9595	0.7910	0.4546	0.1564	0.0446	0.0814	0.2292	0.4591	0.6954
100	Score	0.9612	0.7378	0.3319	0.0750	0.0651	0.2276	0.5131	0.7890	0.9465
	Wald	0.9884	0.8882	0.5468	0.1856	0.0465	0.1005	0.3030	0.5954	0.8423
200	Score	0.9996	0.9757	0.6977	0.1775	0.0556	0.3102	0.7294	0.9599	0.9979
	Wald	0.9998	0.9909	0.8137	0.2988	0.0501	0.1901	0.5836	0.9112	0.9923

Table 4. Empirical type I error rates (under line) and powers of tests for IG(27, 1), $\theta = 0.20$.

n	Method	θ_0								
		0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23	0.24
25	Score	0.0531	0.0332	0.0383	0.0683	0.1146	0.1693	0.2406	0.3386	0.4419
	Wald	0.3825	0.2363	0.1356	0.0727	0.0421	0.0319	0.0382	0.0617	0.0920
50	Score	0.2990	0.1337	0.0525	0.0418	0.0782	0.1537	0.2766	0.4017	0.5590
	Wald	0.6224	0.4029	0.2099	0.0950	0.0446	0.0501	0.0924	0.1523	0.2578
75	Score	0.5438	0.2697	0.0988	0.0428	0.0677	0.1550	0.3047	0.4879	0.6761
	Wald	0.7710	0.5278	0.2700	0.1097	0.0497	0.0571	0.1323	0.2445	0.4195
100	Score	0.7210	0.4130	0.1611	0.0571	0.0616	0.1630	0.3386	0.5600	0.7552
	Wald	0.8735	0.6327	0.3462	0.1282	0.0485	0.0642	0.1731	0.3494	0.5535
200	Score	0.9734	0.7889	0.4010	0.1028	0.0545	0.2039	0.4865	0.7950	0.9462
	Wald	0.9889	0.8803	0.5573	0.1895	0.0495	0.1178	0.3402	0.6589	0.8870

Table 5. Empirical type I error rates (under line) and powers of tests for IG(18, 1), $\theta = 0.25$.

n	Method	θ_0								
		0.21	**0.22**	**0.23**	**0.24**	**0.25**	**0.26**	**0.27**	**0.28**	**0.29**
25	Score	0.0352	0.0382	0.0506	0.0758	0.1106	0.1547	0.2170	0.2716	0.3413
	Wald	0.2629	0.1754	0.1032	0.0624	0.0429	0.0357	0.0362	0.0456	0.0604
50	Score	0.1462	0.0732	0.0422	0.0469	0.0812	0.1281	0.2077	0.2975	0.4174
	Wald	0.4259	0.2649	0.1522	0.0771	0.0481	0.0400	0.0582	0.1007	0.1570
75	Score	0.2985	0.1510	0.0655	0.0419	0.0643	0.1278	0.2174	0.3480	0.4903
	Wald	0.5519	0.3563	0.1932	0.0920	0.0489	0.0548	0.0866	0.1552	0.2465
100	Score	0.4550	0.2387	0.1030	0.0483	0.0624	0.1222	0.2478	0.3960	0.5687
	Wald	0.6750	0.4437	0.2388	0.1007	0.0471	0.0572	0.1131	0.2112	0.3529
200	Score	0.8313	0.5555	0.2471	0.0767	0.0538	0.1425	0.3322	0.5845	0.7888
	Wald	0.9061	0.7046	0.3856	0.1425	0.0529	0.0780	0.2056	0.4227	0.6513

Table 6. Empirical type I error rates (under line) and powers of tests for IG(13.11, 1), $\theta = 0.30$.

n	Method	θ_0								
		0.26	**0.27**	**0.28**	**0.29**	**0.30**	**0.31**	**0.32**	**0.33**	**0.34**
25	Score	0.0324	0.0431	0.0594	0.0784	0.1094	0.1444	0.1834	0.2214	0.2787
	Wald	0.1879	0.1306	0.0915	0.0557	0.0429	0.0328	0.0308	0.0345	0.0457
50	Score	0.0858	0.0535	0.0427	0.0534	0.0736	0.1136	0.1653	0.2352	0.3164
	Wald	0.2997	0.2058	0.1178	0.0717	0.0481	0.0410	0.0513	0.0759	0.1034
75	Score	0.1832	0.0934	0.0534	0.0426	0.0623	0.1103	0.1775	0.2544	0.3573
	Wald	0.4103	0.2550	0.1497	0.0818	0.0511	0.0474	0.0676	0.1048	0.1649
100	Score	0.2806	0.1494	0.0678	0.0442	0.0603	0.1024	0.1809	0.2853	0.4150
	Wald	0.5001	0.3153	0.1690	0.0878	0.0497	0.0474	0.0816	0.1320	0.2227
200	Score	0.6430	0.3718	0.1668	0.0652	0.0517	0.1115	0.2307	0.4030	0.5970
	Wald	0.7682	0.5246	0.2832	0.1242	0.0513	0.0618	0.1361	0.2686	0.4456

Table 7. Empirical type I error rates (under line) and powers of tests for IG(10.16, 1), $\theta = 0.35$.

n	Method	θ_0								
		0.31	**0.32**	**0.33**	**0.34**	**0.35**	**0.36**	**0.37**	**0.38**	**0.39**
25	Score	0.0390	0.0437	0.0677	0.0855	0.1105	0.1273	0.1599	0.2032	0.2378
	Wald	0.1433	0.1077	0.0822	0.0600	0.0435	0.0375	0.0335	0.0334	0.0375
50	Score	0.0618	0.0454	0.0413	0.0535	0.0700	0.1081	0.1438	0.1948	0.2392
	Wald	0.2292	0.1512	0.0991	0.0653	0.0463	0.0434	0.0444	0.0610	0.0761
75	Score	0.1205	0.0697	0.0464	0.0443	0.0619	0.0969	0.1373	0.1968	0.2719
	Wald	0.3080	0.1971	0.1226	0.0778	0.0496	0.0471	0.0497	0.0735	0.1128
100	Score	0.1849	0.1015	0.0578	0.0479	0.0591	0.0903	0.1488	0.2144	0.3023
	Wald	0.3723	0.2356	0.1424	0.0845	0.0518	0.0478	0.0617	0.0976	0.1474
200	Score	0.4521	0.2650	0.1271	0.0573	0.0488	0.0859	0.1653	0.2893	0.4324
	Wald	0.6052	0.4072	0.2285	0.1079	0.0524	0.0541	0.0913	0.1750	0.2854

The histogram, density plot, Box and Whisker plot and the inverse gamma quantile-quantile (Q-Q) plot are shown in Figure 1. They confirmed that the fitted distribution for annual rainfall amounts are not symmetric distribution. Table 8 reports the Akaike information criterion (AIC) [16] results to check the fitting of the distribution for the annual rainfall amounts in Lampang. AIC is defined as $AIC = -2\ln L + 2k,$ where L is the likelihood function and k is the number of parameters. The results show that the annual rainfall amounts in Lampang had an inverse gamma distribution because the AIC value of the inverse gamma distribution was smallest. However, the AIC values of the normal and the inverse gamma distributions are similar but the inverse gamma distribution is more suitable. The reason is that the annual rainfall amounts are the positive values. The annual rainfall amounts in Lampang had an inverse gamma distribution with a shape parameter, $\hat{\alpha} = 20.4747$ and a scale parameter, $\hat{\beta} = 23146.24$, while the estimator of the CV is $\hat{\theta} = 0.2327$ using the maximum likelihood estimation.

Our interest was in testing the population CV of the annual rainfall amounts in Lampang. Suppose the researcher wanted to test the claim that a population CV equals 0.25. The null and alternative hypotheses are respectively given as follows: $H_0 : \theta = 0.25$ versus $H_1 : \theta \neq 0.25.$

The lower and upper critical values of both test statistics were shown in Table 9. The null hypothesis H_0 was not rejected since $0.1509 \leq \theta_0 \leq 0.2992$ and $0.1831 \leq \theta_0 \leq 0.3747$ using test statistics based on the Score and Wald methods, respectively. We conclude that the population CV of the annual rainfall amounts in Lampang does not differ from 0.25 at the 0.05 significance level. Namely, the population standard deviation of annual rainfall amounts is around 0.25 times the population mean.

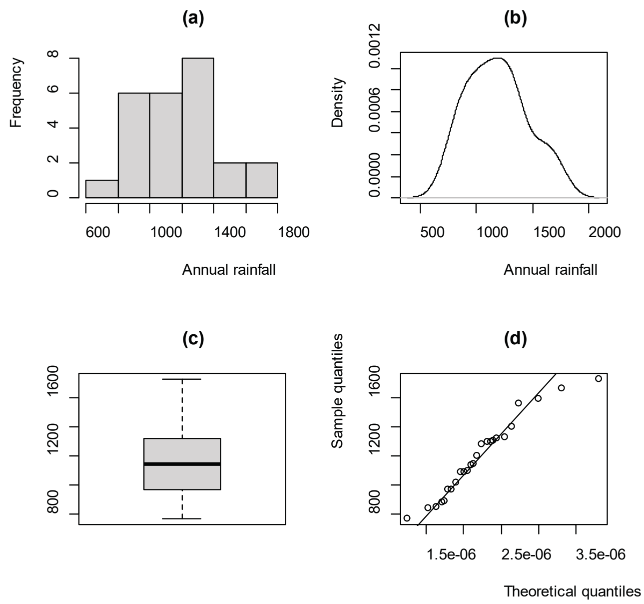

Figure 1. (a) histogram (b) density plot (c) Box and Whisker plot (d) inverse gamma Q-Q plot of the annual rainfall amounts in Lampang, Thailand

Table 8. Results of AIC for the annual rainfall amounts in Lampang, Thailand

Normal	Cauchy	Exponential	Weibull	Gamma	Inverse Gamma
353.3512	419.6644	470.7903	407.9166	405.7664	352.4116

Table 9. Critical values of test statistic based on the score and Wald methods at the 0.05 significance level

Method	Critical values	
	Lower	Upper
Score	0.1509	0.2992
Wald	0.1831	0.3747

6. Discussion and Conclusions

The aim of this study is to identify potential methods that can be recommended to practitioners for testing the CV in an inverse gamma distribution. A general pattern was observed (as expected); as the sample size increased, the power of the test also increased and the empirical type I error rates approached 0.05. Moreover, the power increased as the value of CV departed from the hypothesized value of the CV. It can be observed that for large sample sizes, the performance of both methods did not differ greatly in terms of the power and attaining the nominal size of the test. However, a significant difference was observed for small sample sizes. In addition, the researchers can applied the proposed methods for testing the population CV in an inverse gamma distribution with other data sets fitted well to an inverse gamma distribution. For example, the inverse gamma distribution has been used for the hitting time distribution of a Wiener process. Future research could focus on the one-tailed hypothesis testing.

In this study, two statistical methods for testing the population CV in an inverse gamma distribution were derived. Based on the simulation results, it is evident that the Wald method performed better than the score method in terms of the empirical type I error rate. The score method performed well in the sense of the power of the test when the population CV was smaller than the hypothesized value of the CV. On the other hand, the Wald method performed better when the population CV was greater than the hypothesized value of the CV. In summary, we would recommend the Wald method for testing since its empirical type I error rate is close to the nominal significance level.

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