การสร้างตัวแบบเชิงคณิตศาสตร์ของการตอบสนองทางสภาพต้านทานแบบแมกนิโทเมตริก จากตัวกลางนำไฟฟ้าสามชั้นด้วยชั้นขอบเขตแบบเลขชี้กำลัง

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บทคัดย่อ

ผลเฉลยเชิงวิเคราะห์ของสนามแม่เหล็กในสภาวะคงตัวอันเป็นผลมาจากแหล่งกำเนิดไฟฟ้ากระแสตรงได้ถูกนำเสนอขึ้นสำหรับปัญหาของแหล่งจ่ายไฟฟ้าและตัวรับสัญญาณซึ่งถูกฝังอยู่ในพื้นผิวดินที่มีลักษณะเป็นชั้นขนานกับแนวระดับ ตัวแบบของพื้นโลกที่มีลักษณะเป็นชั้นแบบ 3 ชั้นต่อเนื่องกันถูกนำมาพิจารณาในกรณีที่พื้นผิวดินชั้นบนและชั้นล่างเป็นแบบเอกพันธุ์ โดยที่ชั้นกลางเป็นชั้นรอยต่อซึ่งมีสภาพนำไฟฟ้าเปลี่ยนแปลงแบบเลขชี้กำลังตามความลึกของพื้นโลก การแปลงฮันเกลแบบทั่วไปถูกนำมาใช้ในปัญหาซึ่งทำให้ได้ผลเฉลยเชิงวิเคราะห์ ผลลัพท์ที่เกิดขึ้นถูกนำมาเขียนกราฟแสดงพฤติกรรมของสนามแม่เหล็ก แนวเส้นโค้งของสนามแม่เหล็กแสดงถึงนัยสำคัญบางประการของการแปรผันของสภาพนำไฟฟ้า

คำสำคัญ: สภาพต้านทานแบบแมกนิโทเมตริก, ชั้นขอบเขตแบบเลขชี้กำลัง, การแปลงฮันเกล

Mathematical Modelling of Magnetometric Resistivity Response from a Three-Layered Conductive Medium with Exponential Boundary Layer

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Abstract

An analytical solution of the steady state magnetic field resulting from a direct current source is derived for the problem of a buried current source and a buried receiver within a horizontally stratified layered earth structure. Three-layered continuously earth model is considered in the case of homogeneous overburden and host medium in which the middle is a boundary layer whose conductivity varies exponentially with depth. The generalized Hankel transform is introduced to our problem and analytical result is obtained. The effects of magnetic fields are plotted to show the behavior of the fields. The curves of magnetic fields show some significance to the variation of conductivity.

Keywords: magnetometric resistivity, exponential boundary layer, Hankel transform

1. Introduction

Many authors have investigated the nature of the resistivity response resulting from a horizontally stratified layered earth containing transitional layers. Mallick and Roy (1968) firstly considered the resistivity sounding on a two-layer earth with transitional boundary. Lal (1970), Jain (1972), Koefoed (1979) studied the problem of a three-layered earth model containing transitional layers. For the particular case of an exponentially varying conductive layer, Banerjee et al. (1980), Kim and Lee (1996) discussed the problem of a multilayered earth structure and derived the specific case for a three-layered model. Juagwon and Sripanya (2021) solved the problem with a buried electrode of resistivity sounding in a three-layered medium containing transitional layers.

In this paper, we derive an analytical solution of the steady state magnetic field resulting from a direct current source buried within a three-layered earth structure. The overburden and host medium have constant conductivities, whereas the middle layer has an exponentially varying conductivity with depth. The generalized Hankel transform is introduced to our problem and analytical result is obtained.

2. Magnetic Field from a DC Source in a 1D Structure

Let us consider a geometric model of the earth's structure described and discussed by Sato (2000). The earth's structure consists of two conductive half-spaces (see Figure 1). The half-space above the ground surface $\left(z<0\right)$ is a region of air, denoted by layer 0. The half-space below the ground surface $\left(z>0\right)$ is a three-layered horizontally stratified earth with depth to the layers h_1, h_2 measured from the ground surface (the lowermost layer extending to infinity). A point source of direct current I is deliberately located at the interface $z=h_1$ of layer 1 and layer 2 to simplify the mathematics. Each layer has conductivity as a function of depth, i.e., $\sigma_k\left(z\right)$ for layer $0 \le k \le 3$. The general steady state Maxwell's equations in the frequency domain (Edwards, 1988) can be used to determine the magnetic field for this problem. Since the problem is axisymmetric and the vector magnetic field has only the azimuthal component in cylindrical coordinates, for simplicity, we use $\tilde{H}$ to represent the azimuthal component in the following derivations (Sripanya, 2016), and we now have

$$\frac{\partial^2 \tilde{H}}{\partial z^2} + \sigma \frac{\partial}{\partial z}\left(\frac{1}{\sigma}\right)\frac{\partial \tilde{H}}{\partial z} - \lambda^2 \tilde{H} = 0\,, \tag{1}$$

where λ is the scaling factor. The magnetic field in each layer can be obtained by taking the inverse generalized Hankel transform (Ali and Kalla, 1999) to the solution of equation (1). The problem satisfies the physical boundary conditions presented by Sripanya (2016).

2.1 Homogeneous Layer

For each layer k, where $0 \le k \le 3$ and $k \ne 2$, having constant conductivity (σ_k is positive constant), equation (1) reduces to

$$\frac{\partial^2 \tilde{H}_k}{\partial z^2} - \lambda^2 \tilde{H}_k = 0 \tag{2}$$

and the solution is

$$\tilde{H}_k\left(\lambda, z\right) = A_k e^{-\lambda z} + B_k e^{\lambda z}\,, \tag{3}$$

where the unknown coefficients A_k and B_k are arbitrary constants, which can be determined by using the boundary conditions (see Sato, 2000).

2.2 Exponential Boundary

For an exponentially varying conductive layer, the variation in conductivity of layer 2 is denoted by

$$\sigma_2\left(z\right) = c\exp\left(b\left(z-h_1\right)\right), \tag{4}$$

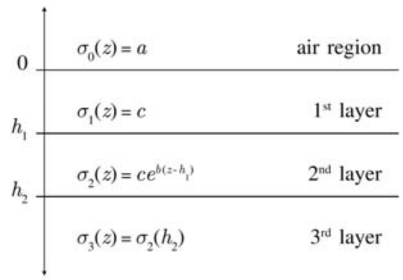

Figure 1 Geometric model of three-layered earth structure with exponential boundary.

where b, c and h_1 are constants that preserve $\sigma_2\left(z\right) > 0$. Hence, the magnetic field in an exponentially varying conductive ground can be written as

$$\tilde{H}_2\left(\lambda, z\right) = A\exp\left(\left(z-h_1\right)\xi\right) + B\exp\left(\left(z-h_1\right)\zeta\right), \tag{5}$$

where ξ and ζ are given by

$$\xi = \frac{1}{2}\left(b - \left(b^2 + 4\lambda^2\right)^{1/2}\right) \quad \text{and} \quad \zeta = \frac{1}{2}\left(b + \left(b^2 + 4\lambda^2\right)^{1/2}\right). \tag{6}$$

The unknown coefficients A and B are arbitrary constants, which can be determined by using the boundary conditions.

3. Solution of the Problem

3.1 3-layered Earth

Consider a 3-layered earth model with a nonconductive layer 0 (representing a region of air whose conductivity a is approximately equal to zero). An overburden has a constant conductivity c with thickness h_1 overlying an exponential boundary layer having continuously varying conductivity σ_2, as given above, with thickness h_2, whereas a host medium has a constant conductivity $\sigma_2\left(h_2\right)$ with infinite depth. The buried current source and receiver are located at the interface $z = h_1$. The magnetic field in an exponentially varying conductive medium can be written as

$$H\left(r, z\right) = \int_0^\infty \left(\alpha\exp\left(\left(z-h_1\right)\xi\right) + \beta\exp\left(\left(z-h_1\right)\zeta\right)\right) J_1\left(\lambda r\right) d\lambda\,, \tag{7}$$

where J_1 is the Bessel function of the first kind of order one, and

$$\alpha = \upsilon/\varphi\,, \quad \beta = \omega/\varphi\,, \quad \upsilon = \varepsilon_\zeta\left(1+\varepsilon_\varsigma\right)\left(b^2 + b\delta + 2\lambda\left(2\lambda+\delta\right)\right)I\,,$$

$$\omega = -\varepsilon_\xi\left(1+\varepsilon_\varsigma\right)\left(-b^2 + b\delta + 2\lambda\left(-2\lambda+\delta\right)\right)I\,, \quad \varphi = 4\pi\delta\left(b\left(\varepsilon_\zeta - \varepsilon_\xi\right) + \varepsilon_\varsigma\left(\varepsilon_\xi\left(-2\lambda+\delta\right) + \varepsilon_\zeta\left(2\lambda+\delta\right)\right)\right),$$

$$\varepsilon_\varsigma = \exp\left(2\lambda h_1\right), \quad \varepsilon_\xi = \exp\left(\delta h_1\right), \quad \varepsilon_\zeta = \exp\left(\delta h_2\right), \quad \delta = \left(b^2 + 4\lambda^2\right)^{1/2}.$$

3.2 Uniform Half-space

In the case of a uniform half-space, the magnetic field as shown in equation (7) can be determined by setting b equal to zero and we obtain

$$H\left(r,z\right)=\int_0^\infty\left(\cosh\left(\lambda h_1\right)\exp\left(-\lambda z\right)I\Big/2\pi\right)J_1\left(\lambda r\right)d\lambda\,, \tag{8}$$

which is the same result obtained by Sripanya (2016).

4. Numerical Experiments

In our sample tests, we calculate the magnetic fields resulting from the models of practical interest. Both of the example models are 3-layered electrically conductive earth structures with a nonconductive layer 0, as shown in Section 3. These models are referred to as Models 1 and 2, which are given in equations (7) and (8), respectively. The values of the model parameters are tabulated in Table 1. Chave's algorithm (1983) is used for numerically calculating the inverse Hankel transform of the magnetic field solutions. The electric current of 1 ampere is used in our computations. The results of our models are plotted and compared to show the behavior of magnetic fields against source-receiver spacing r at different depths $z=5,5.5,\ldots,15$ metres as shown in Figure 2. We see that the curves of magnetic fields obtained from the first model are quite different from the second model. The contrast of electrical conductivity gives effect to the magnitude of magnetic field. It can be seen that the magnitude of magnetic field from the first model is much higher than the magnetic field from the second model at the same depth and source-receiver spacing. We observe that if the conductivity of a boundary layer is high, it will lead to the large sized magnitude of magnetic field. This means that the differences of magnetic curves are depended on the variation of conductivity.

5. Conclusions

We have derived an analytical solution of the steady state magnetic field for the problem of source and receiver electrodes buried within a three-layered earth structure with homogeneous overburden and host medium in which the middle is a boundary layer whose conductivity varies exponentially with depth. The generalized Hankel transform is introduced to our problem and analytical result is obtained. The effects of magnetic fields are plotted and compared to show the behavior in response to different ground structures at many depths while some parameters are approximately given. The curves of magnetic fields show some significance to the variation of conductivity.

Table 1 Model parameters used in our sample tests.

Model	c (S/m)	b (m^{-1})	h_1 (m)	h_2 (m)
1	0.05	0.25	5	15
2	0.05	0	5	15

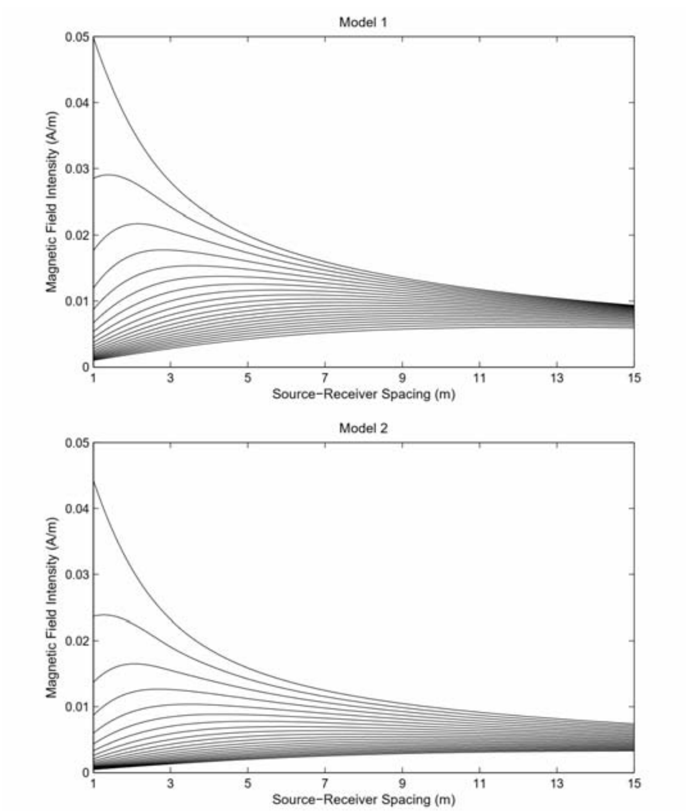

Figure 2 Behavior of magnetic fields from our example models.

6. References

Ali, I., & Kalla, S. (1999). A generalized Hankel transform and its use for solving certain partial differential equations. **J. Austral. Math. Soc. Ser. B.**, 41 (1), 105-117.

Banerjee, B., Sengupta, B. J., & Pal, B. P. (1980). Apparent resistivity of a multilayered earth with a layer having exponentiality varying conductivity. **Geophys. Prosp.**, 28 (3), 435-452.

Chave, A. D. (1983). Numerical integration of related Hankel transforms by quadrature and continued fraction expansion. **Geophysics.**, 48 (12), 1671-1686.

Edwards, R. N. (1988). A downhole magnetometric resistivity technique for electrical sounding beneath a conductive surface layer. **Geophysics.**, 53 (4), 528-536.

Jain, S. C. (1972). Resistivity sounding on a three-layer transitional model **Geophys. Prosp.**, 20 (2), 283-292.

Juagwon, K. and Sripanya, W. (2021). Mathematical modelling of buried electrode resistivity response from a three-layered conductive medium containing transitional layers. **J. Dyn. Syst. Geom. Theor.**, (Accepted).

Kim, H.-S., & Lee, K. (1996). Response of a multilayered earth with layers having exponentially varying resistivities. **Geophysics.**, 61 (1), 180-191.

Koefoed, O. (1979). Resistivity sounding on an earth model containing transition layers with linear change of resistivity with depth. **Geophys. Prosp.**, 27 (4), 862-868.

Lal, T. (1970). Apparent resistivity over a three layer earth with an inhomogeneous interstratum. **Pure Appl. Geophys.**, 82 (1), 259-269.

Mallick, K., & Roy, A. (1968). Resistivity sounding on a two-layer earth with transitional boundary. **Geophys. Prosp.**, 16 (4), 436-446.

Sato, H. K. (2000). Potential field from a dc current source arbitrarily located in a nonuniform layered medium. **Geophysics.**, 65 (6), 1726-1732.

Sripanya, W. (2016). Mathematical modelling of magnetic field from a DC source in a multilayered medium. **J. Interdiscip. Math.**, 19 (5&6), 1015-1023.